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MODE MODULATION IN OPTICAL FIBRE TRANSMISSION. CONVERSION OF ANALOG SIGNALS INTO VARIABLE MODE STRUCTURE

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Abstract—A way to extend the applications of fibre-optical multimode transmission systems is based on the nature of light propagation in multimode waveguides, namely, propagation in the form of discrete modes and modal dispersion. The suggestion is to transform the initial signal into a distribution of the modes excited in the fibre. The signal carried by each mode is viewed as a discrete two-state Markov process.

The maximum distance of transmission is estimated under mode coupling which is intensified owing to fibre bends. The conditions are evaluated that allow for a correct solution of the maximum transmission length problem. The advantages of time-domain analysis are demonstrated for a signal transmitted as a train of pulses of identical length and amplitude, exciting different numbers of modes.

The practical coupling of multimode optical fibres in optical transmission systems has stimulated research into new capabilities of such systems, transmission reliability and noise immunity. This paper considers a method of expanding the functional capabilities of multimode optical fibre transmission systems. The method harnesses the natural features of electromagnetic wave propagation in multimode waveguides—the discrete nature of light propagation in waveguides and modal dispersion.

The propagation of light in optical fibre systems in the form of discrete modes has attracted some attention mainly in connection with signal multiplexing by separate modulation of individual modes with subsequent independent detection (see, e.g. Cozanne *et al.* [1]). On the other hand, however, the information in a signal travelling in a lightguide may be judged by analysis of the mode structure supported by the fibre. The conceptual framework of this method has been outlined in a study of sensitivity of amplitude transducers based on multimode graded-index lightguides [2].

The following approach is based on the transformation of an initial signal into a distribution of modes launched in the fibre. Assume that each mode of order n carries an energy $G_n(t, z)$, and at the fibre input ($z = 0$) where $G_n(t, 0)$ there are two levels—zero and one. A simple example of such a transformation may be the linear transformation of a normalized positive analog signal $g(t)$ into a number of modes launched into a lightguide, *viz.*,

$$G_n(t, 0) = \begin{cases} 1, & g(t) \geqslant (n-1)/(N-1), \\ 0, & g(t) < (n-1)/(N-1), \end{cases} \tag{1}$$

that is, when $g(t) = g(t)_{\max} \equiv 1$, the total number of modes N is launched into the fibre, and when $g(t) = g(t)_{\min} \equiv 0$ only one mode of the lowest order is excited, and so on. This transformation can be realized by making the angle of convergence of the illuminating beam vary in proportion with the magnitude of the signal.

The output signal of each mode will be considered as a two-state Markov process, and the inverse transformation will be performed by analysing the energy levels carried by the modes at the fibre output. For a transformation (1) in particular, a trivial inverse transformation may consist in evaluating the angle at which the radiation leaves the fibre assuming that there this angle is directly related to the mode order.

The capabilities of transmission by varying the structure of modes launched in a fibre are controlled primarily by the effection mode coupling length. If bends in the fibre do not affect attenuation and do not substantially alter the distribution of the effective refractive index, then to a first approximation the transmission distance will be governed by the fabrication technology of multimode graded-index fibres and can at present reach several kilometres [3].

Let us consider the limitations that can occur owing to mode coupling caused by external impacts. Fibre bends described by a function $f(z)$ are known to cause mode conversion [2], i.e. energy launched into an initial mode of order n and propagation constant β_n being coupled to modes of order m so that the average propagation constant of the output beam becomes

$$\langle \beta \rangle = \beta_n + \nu \tag{2}$$

where

$$\nu = |d|^2,$$

$$d(z) = \mathrm{i}\sqrt{k/\omega} \int_0^z f(\tau)\,\mathrm{e}^{\mathrm{i}w\tau}\,\mathrm{d}\tau.$$

Mode coupling is most effective for modes whose propagation constants satisfy the condition

$$|\beta_n - \beta_m| = 2\pi\omega \tag{3}$$

(the graded-index fibre is assumed to be axially uniform).

The coefficients of mode coupling for arbitrary function $f(z)$ are symmetric in indexes m and n and can be expressed in terms of generalized Laguerre polynomials [4]

$$W_m^n = \frac{l_1!}{l_2!} \nu^{|m-n|} l^{-\nu} |L_{l_1}^{|m-n|}(\nu)|^2, \tag{4}$$

where $l_1 = \min(m, n)$ and $l_2 = \max(m, n)$.

Suppose that the analog signal at the fibre output is recovered by analysing the field at the fibre end at every instant of time (provided the characteristic time of signal variation $\tau_s \gg \beta_n z$ for any n, and z is such that $0 \leqslant z \leqslant L_0$, where L_0 is the total length of the fibre).

We define the mode structure of a coherent field at the input and output of the fibre by the complex amplitudes

$$\xi_{\mathrm{in}}(t, \mathbf{x}) = \sum_1^N \xi_n(t)\psi_n(\mathbf{x})_{z=0},$$
$$\xi_{\mathrm{out}}(t, \mathbf{x}) = \sum_1^N \xi_n(t)\psi_n(\mathbf{x})_{z=L_0} \tag{5}$$

with

$$|\xi_n(t)_{z=0}|^2 = G_n(t, 0)$$

and

$$|\xi_n(t)_{z=L_0}|^2 = G_n(t, L_0),$$

where $\{\psi_n(\mathbf{x})\}$ is the system of basis functions corresponding to transverse-mode configurations.

In a first approximation,

$$G_n(t, L_0) \approx \left[G_n(t, 0) + \sum_{m=1}^N W_m^n(t) G_m(t, 0) - \sum_{m=1}^N W_n^m(t) G_n(t, 0) \right] k_\alpha, \tag{6}$$

where k_α is the transmittance coefficient for the signal in the fibre.

For any specific function $f(z)$ describing bends of the axis, the maximum transmission distance L_0 that provides an adequate transmission of $\{G_n(t, 0)\}$ into $\{G_n(t, L_0)\}$ may be determined by the relations (2), (4) and (6).

In a more general formulation, the maximum transmission length may be evaluated by optimizing the transformation of a signal $g(t)$ into $\{G_n(t, 0)\}$, i.e. by seeking an optimal distribution of information in $g(t)$ over the modes of the fibre. This problem should be solved after a preliminary investigation into a possible class of functions and, above all, into the feasibility of the physical and engineering implementation of the translation of $g(t)$ into $\{G_n(t, 0)\}$. For arbitrary functions $f(z)$, one may make good use of the known estimates [2] of mode conversion due to the resonance bending of the fibre axis with a period of oscillation of paraxial rays, $F(z) = A \sin \omega z$. For $\omega z \geqslant 1$, i.e. at distance greater than the period of oscillation,

$$d(z) = \frac{Az}{2}\sqrt{\frac{k}{2\omega}}, \tag{7}$$

where k is the wavenumber, and for the maximum length preserving the state $G_n(t, z) = 1$ we obtain the estimate

$$L_{0n} \approx \frac{2}{A}\sqrt{\frac{2\omega}{k(2n+1)}}. \tag{8}$$

Thus, if the function contains a continuous set of independent spatial frequency components (i.e. if $f(z)$ describes the effect of white noise), the transformation of a signal into a number of launched modes (distribution of information over modes) must be nonuniform in agreement with the nonlinear dependence of the maximum length of effective mode coupling upon the number of a mode, as defined by Eq. (8).

There exist additional capabilities of the mode modulation method that can be realized by harnessing the effect of modal dispersion which is traditionally viewed only as a factor contributing to the attenuation of multimode fibre transmission lines.

Assume that the impulse response of a fibre to a full mode structure launched into it is described by the function $h_N(t - t_0)$ of width τ_N. Let the signal be a train of pulses of duration $\tau_0 \ll \tau_N$ arriving with a period $T_0 > \tau_N$. Each pulse launches a number of modes $n(t)$ defined by (1). The duration of a signal at the fibre output will be governed by the dispersion of the initial pulse, which will be the sum of the dispersions of the initial distribution of modes and the modes excited inside the fibre [1]. The above estimate of mode coupling length holds here as well, but the length of mode interaction is by no means the maximum attainable transmission length. In contrast to the earlier situation when the detected signal is identified by the presence or absence of a given level of launching of a particular mode (spatial analysis of the output field of the fibre), analysis in time domain can be used by determining the width of the impulse response $\tau_\Sigma(n)$ to a unit pulse that has initially launched n modes. At a distance equal to the length of total effective mode coupling length (equilibrium distribution of modes), the spatial analysis defies a direct evaluation of the type of mode launched initially. However the impulse response function contains the required information since

$$\tau_\Sigma(n) \approx \tau_n + \sum_i \Delta\beta_{ni} z_i, \tag{9}$$

where τ_n is the duration of the impulse response corresponding to dispersion of n modes, z_i is the length corresponding to the propagation of initially absent mode i, and $\Delta\beta_{ni}$ is the difference of the propagation constants for modes n and i (for simplicity we assume that these modes carry equal energy).

Thus, under an equilibrium distribution of modes the length of response τ_Σ is controlled not only by the conditions of propagation but also by the initial mode structure. It varies from

$$\tau_\Sigma^{\min} = \sum_{i=1}^{N} \Delta\beta_{ni} z_i \qquad \text{for} \quad n = 1$$

to

$$\tau_\Sigma^{\max} = \tau_N \qquad \text{for} \quad n = N, \tag{10}$$

i.e. the maximum transmission length can substantially exceed the length of effective mode coupling (for example, for $\tau_\Sigma^{\min} \ll \tau_\Sigma^{\max}$).

The analysis of the initial structure of launched modes by measuring $\tau_\Sigma^{(t)}$ offers another valuable advantage over spatial analysis, namely, a higher stability with respect to variation of loss level in the fibre, which does not relate to the variation of mode coupling, for in this case the function $h_\Sigma(t)$ varies in magnitude but not in length $\tau_\Sigma(t)$.

The above estimates cannot, of course, pretend to completely analyse the capabilities of the mode modulation method, however they are sufficient to yield some preliminary conclusions.

First, there exists the possibility of adequate transformation and transmission of signals by the method discussed in this paper, provided that the transformation of signals in modes, say quantization of the initial analog signal into a number of modes in agreement with the condition (1), meets the general requirements on transmission quality (we note that the number of modes, i.e. quantization levels, runs into several hundreds in commercial multimode fibres). The transmission length is controlled by the effective length of mode coupling which, when mode conversion intensifies as a result of microbending of the fibre, is in turn defined by the function describing the bending.

Second, a correct definition of the maximum transmission distance requires that a rule should be defined for transformation of a function $g(t)$ into the distribution $\{G_n(t, 0)\}$. Also, a criterion should be adopted to check how adequately the distribution $\{G_n(t, 0)\}$ is transformed in the distribution $\{G_n(t, L_0)\}$; depending on the requirements imposed on the communication channel, this criterion may be a specified probability of error, a minimum signal-to-noise ratio for each G_n, and a correlation of the distributions $\{G_n(t, 0)\}$ and $\{G_n(t, L_0)\}$. It is worth noting that the transformation of a signal into a mode distribution should take account of the different sensitivity of the modes to external disturbances, both correlated and uncorrelated, i.e. the signal information should in general be distributed nonuniformly among the modes with the condition that the probabilities of error in the received amounts of information transmitted are the same in the different modes.

Third, the integral variation of the loss level k_α in a fibre, originating from other sources than mode convergence, affects the signal in a manner different from the traditional methods of analog modulation and attenuates all modes in the same proportion. Accordingly, methods of reception may be used that compensate for the variation of k_α to some degree, for instance, by automatic gain control for all mode channels in detection.

The quasi-digital idea underlying this approach deserves appropriate attention. The fibre plays the role of an analog/digital converter that transforms an initial signal into a set of discrete modes or into a pulse of specified length (we note that for a lightguide supporting a few modes this pulse can be converted into a sequence of discrete pulses). The fibre output may be analysed in discrete circuitry available, for example, for measurement of pulse duration and/or counting fibre output pulses, or in circuits with holographic elements for mode structure analysis [4].

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